PhD dissertation subject

Decomposition methods and polyhedral approaches for integer quadratic optimization

Context

The aim of this topic is to propose new solution methods for integer quadratic optimization. These problems can be found in many application domains [5], such as energy, finance, logistics, networks, image processing and computer vision.

These problems are generally NP-hard and solving them exactly, or even obtaining good relaxations, remains a challenge today, especially for instances of realistic size. Unlike integer linear optimization, there is currently no generic solver that can efficiently solve these problems.

Our aim here is to study the structural properties of these problems and solve them using combinatorial optimization techniques.

Study of structural properties by using decomposition methods and polyhedral approaches

There exist various decompositions in the scientific literature adapted to particular cases of the integer quadratic problems that will be considered in this subject, such as, for example, the Danzig-Wolfe decomposition, the Benders decomposition, the Lagrangian decomposition, the clique decomposition, the simplicial decomposition, ... As a first step, we would like to consider the generalization of the BQP (Boolean Quadric Polytope) decomposition recently proposed in [1] to the case where variables are no longer binary, but rather integer in general [3]. A study of the structure of the polyhedra associated with this decomposition will be envisaged in order to attempt to establish theoretical results on the quality of the relaxation, and then to reinforce the quality of the bounds obtained via the generation of reinforcement inequalities.

Implementation of the techniques

These resolution methods will be implemented so as to obtain information on the quality of the bounds obtained. A study of heuristic techniques for obtaining feasible solutions will also be considered. All these approaches will then be integrated into a Branch & Cut & Price-type scheme to provide optimal integer solutions and be compared with existing methods for solving these integer quadratic problems [2, 4]. When solving the subproblems arising from the decompositions, the use of generic commercial solvers, such as Cplex or Gurobi, will be favored in order to preserve the generic aspect of the proposed approach.

The implementation will be carried out in Julia, in order to make the code available to the scientific community.

Advisors

Advisor: Lucas Létocart, PU, LIPN

Co-advisor: Silvia Di Gregorio, MCF, LIPN

The two co-supervisors have complementary expertise in mathematical optimization, particularly in nonlinear optimization as well as in polyhedral approaches and decomposition methods.

Bibliographie :

[1] Bettiol, Enrico and Bomze, Immanuel and Létocart, Lucas and Rinaldi, Francesco and Traversi, Emiliano Mining for diamonds—Matrix generation algorithms for binary quadratically constrained quadratic problems. *Computers & Operations Research,*, 142 (105735), 2022.

[2] Khajavirad, Aida and Sahinidis, Nikolaos V. A hybrid LP/NLP paradigm for global optimization relaxations *Mathematical Programming Computation*, 10:3(383-421), 2018.

[3] Bonami, Pierre and Günlük, Oktay and Linderoth, Jeff Globally solving nonconvex quadratic programming problems with box constraints via integer programming methods *Mathematical Programming Computation*, 10:3(333-382), 2018.

[4] Bonami, Pierre and Biegler, Lorenz.T. and Conn, Andrew R. and Cornuejols, Gérard and Grossmann, Ignacio E. and Laird, Carl D. and Lee, Jon and Lodi, Andrea and Margot, François and Sawaya, Nicolas and Waechter, Andreas An Algorithmic Framework for Convex Mixed Integer Nonlinear Programs *IBM Research Report RC23771*, 2005.

[5] Burer, Samuel and Letchford, Adam N. Non-convex mixed-integer nonlinear programming: A survey *Surveys in Operations Research and Management Science*, 17:2(97-106), 2012.