Optimized Schwarz Methods for Maxwell's equations: Applications to the numerical simulations of meta-surfaces

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Meta-surfaces are thin sheets of material containing periodically spaced small obstacles. Placed into a homogeneous medium, these surfaces may have very interesting physical properties. More specifically, depending on the shape and the nature of the obstacles, these surfaces may behave like a Faraday cage, namely block all the waves (this phenomenon is known as the shielding effect, see [2, 3]). The numerical simulation of these media remains a challenging problem: indeed, the presence of these micro-stuctures (requiring a very refined mesh) combined with the vectorial nature of Maxwell's equations lead to problems of huge size. Therefore, parallel computations are required.

Domain decomposition methods, and, among them, optimized Schwarz methods, provide a powerfull framework for this parallelization. The classical Schwarz algorithm, based on the transmission of Dirichlet traces between subdomains, is known to have poor convergence properties for the computation of harmonic waves. Optimized Schwarz algorithms rely on transmission of Robin data, and can be used with or without overlapping between the subdomains. They improve drastically the convergence. A partial analysis of the problem for the Helmholtz equation in two dimensions can be found in [4]. The extension for the Maxwell system in 2 dimensions based on asymptotic ansatz has been proposed in [5], and experiments have been done in two and three dimensions, for instance in [6]. New results on Helmholtz with a complete analysis of the optimisation problem have been obtained in [1].

The objective of this PhD is to extend these results to the case of the 3D Maxwell's equations with or without absorption, in time domain and in harmonic regime. These methods will be implemented for the simulations of the diffraction by meta-surfaces on highly parallel computers (MAGI/Université Paris 13). For this second part, a particular interest in parallel computing is required.

References

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