PhD Thesis proposal

Schwarz waveform relaxation (SWR) algorithms for optimal control problem constrained by the wave equation

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The PhD thesis deals with optimization problems in which the underlying systems are governed by hyperbolic partial differential equations (PDEs), which model wave-type phenomena. Such problems often arise from inverse problems and data assimilation. One classic example is seismic inversion, where the observed data are the seismic waves reflected by the different subsurface rock formations, and the goal is to deduce and refine the rock parameters that make the observed data possible.

The simulation of such problems (based on the Hilbert Uniqueness Method [1]) requires the computation of backward and forward wave equations, and leads to the numerical solution of large scale problems. For an efficient simulations of them, it is necessary to design parallel in time methods. One way to do it is to use the Schwarz waveform relaxation (SWR) approach, see [2, 3]. The main idea is to decompose first the problem in space to obtain a collection of (coupled) space-time subproblems, then iterate while exchanging interface information over the whole time window.

The objective of this PhD thesis is to design optimized Schwarz waveform relaxation (SWR) algorithms, for optimal control problem constrained by the wave equation. Special care must be given to the discretization, as it produces high frequency waves with slow group velocity which perturb drastically the control process. This has been studied so far only for the semi-discrete one-dimensional wave equation, in [4]-[5]. A ongoing master thesis is extending the results to the fully discrete case. In the next PhD thesis, we will study thethe possibility to cancel high frequency waves by domain decomposition with time windows.

We will consider the linear quadratic optimization problem with regularization coefficients. We will propose approximate transmission conditions, which optimize the forward-backward problem, taking the regularization coefficients into account. We will study the convergence of the algorithm. We will consider the extension of bi-grid filtering to SWR in connection with coarse grid preconditioning in both space and time.

The candidate for the thesis is Vuong Dang Thanh. He was a student in the "Master franco-vietnamien délocalisé en mathématiques appliquées de HCMC". We saw during the course and now during the master thesis "distancielle" that he is knowledgeable, intelligent, and creative. He is certainly a very good candidate for the PhD.
Location: LAGA, Université Paris 13, Villetaneuse, FRANCE. `https://www.math.univ-paris13.fr/laga/index.php/fr/`

References

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