PhD proposal for Nguyen Quynh Huong TRAN

Population replacement strategies in the fight against arboviroses in heterogeneous environment.

Supervisors : Hoang Hung Vo (Saigon University) and Nicolas Vauchelet (Université Paris 13)
Laboratoire Analyse, Géométrie et Applications, UMR 7539
Contact : vhhungkhtn@gmail.com, vauchelet@math.univ-paris13.fr

In the fight against arboviruses like Dengue, Chikungunya, or Zika, several strategies are under investigation. One of these strategies consists in replacing the wild population of Aedes mosquitoes by a population infected by the bacteria Wolbachia. Indeed it has been observed experimentally that individuals infected by this bacteria are not able to transmit those arboviruses to human [2]. Moreover the bacteria is transmitted vertically from mother to offsprings. Then a strategy of control of epidemic consists in releasing Wolbachia infected mosquitoes to replace the existing population. This strategy has been succesfully implemented on the field in [1].

In order to model the spatial propagation of this bacteria into a host population, a mathematical system of reaction-diffusion equations has been proposed in [4]. This system governs the dynamics of the density of the infected population n_1 and the Wolbachia-free population n_2. It reads

$$\partial_t n_i - D\Delta n_i = (1-s_f)F_u n_i\big(1-\frac{N}{K}\big) - \delta d_u n_i,$$
$$\partial_t n_u - D\Delta n_u = F_u n_u(1-s_h p)\big(1-\frac{N}{K}\big) - d_u n_u.$$

In this system we use the following parameters : $d_u, d_i = \delta d_u$ are death rates ($\delta > 1$) ; $F_u, F_i = (1-s_f)F_u$ are the fecondity parameters ; s_h is the cytoplasmic incompatibility parameter (fraction of uninfected females' eggs fertilized by infected males which will not hatch) ; K is the environmental capacity.

The aim of this work is twofold. The first goal consists in focusing on the influence of heterogenity in the environement into the propagation. These heterogeneities may be modelled thanks to the parameter K, by assuming that it is a function of the space variable. A first step in the understanding of this influence has been obtained in [3] on a simpler reduced system. It is expected that spacial heterogeneity may block the spreading of Wolbachia-infected mosquitoes. The second goal is to study the same model as of [3] under the Stefan free boundary setting as of [5]. We would like to explain more precisely this blocking phenomena for the full system, the influences of the heterogeneities and the free boundary on the propagation of Wolbachia.

Bibliographie :

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